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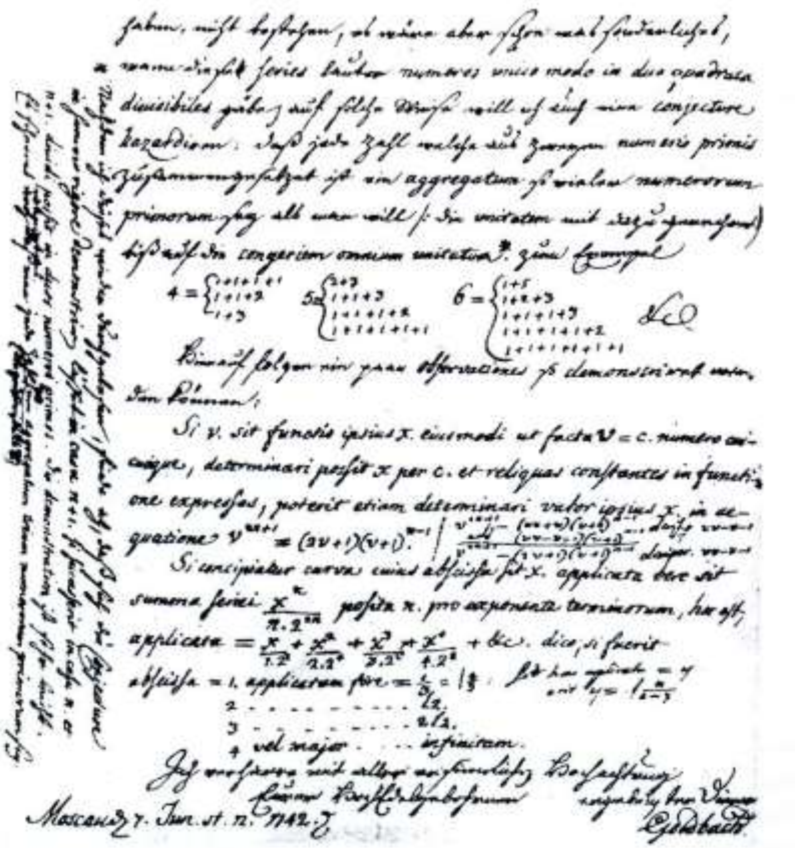
On the Road to Goldbach

Überblick

1. Einführung
2. Primes & Composites
3. Zerlegungen
4. Richtung Goldbach
5. Schlussfolgerung

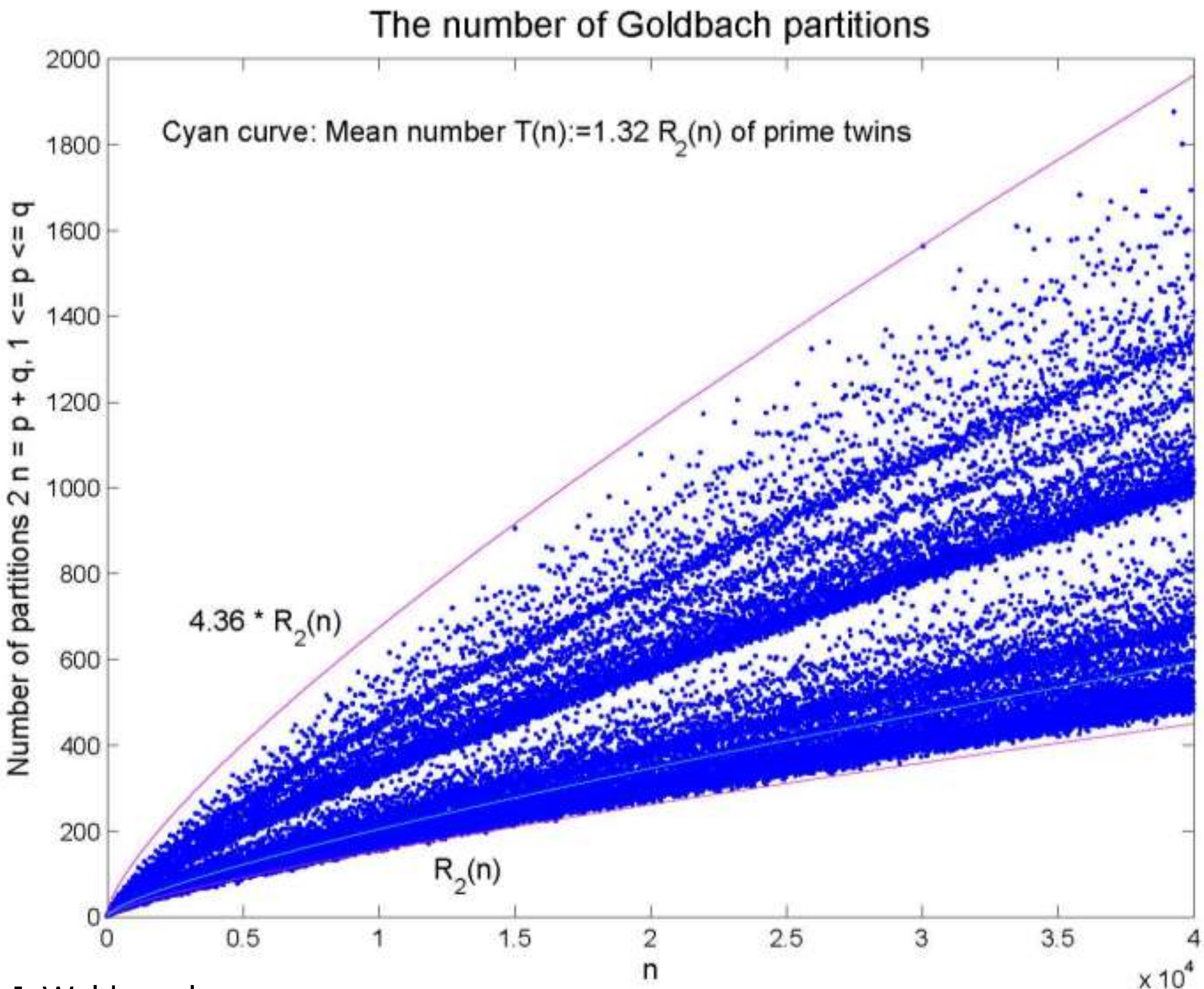
Einführung

- Gesundheitsbedingungen
  - Notwendig:
    1. Körperliche Bewegung
    2. Mentale Bewegung
    3. …
- Goldbach Vermutung
  Jede gerade Zahl >2 lässt sich als Summe von zwei Primzahlen ausdrücken.
- Primzahl
  ist eine natürliche Zahl >1 die nur durch sich selbst und durch 1 ganzzahlig teilbar ist



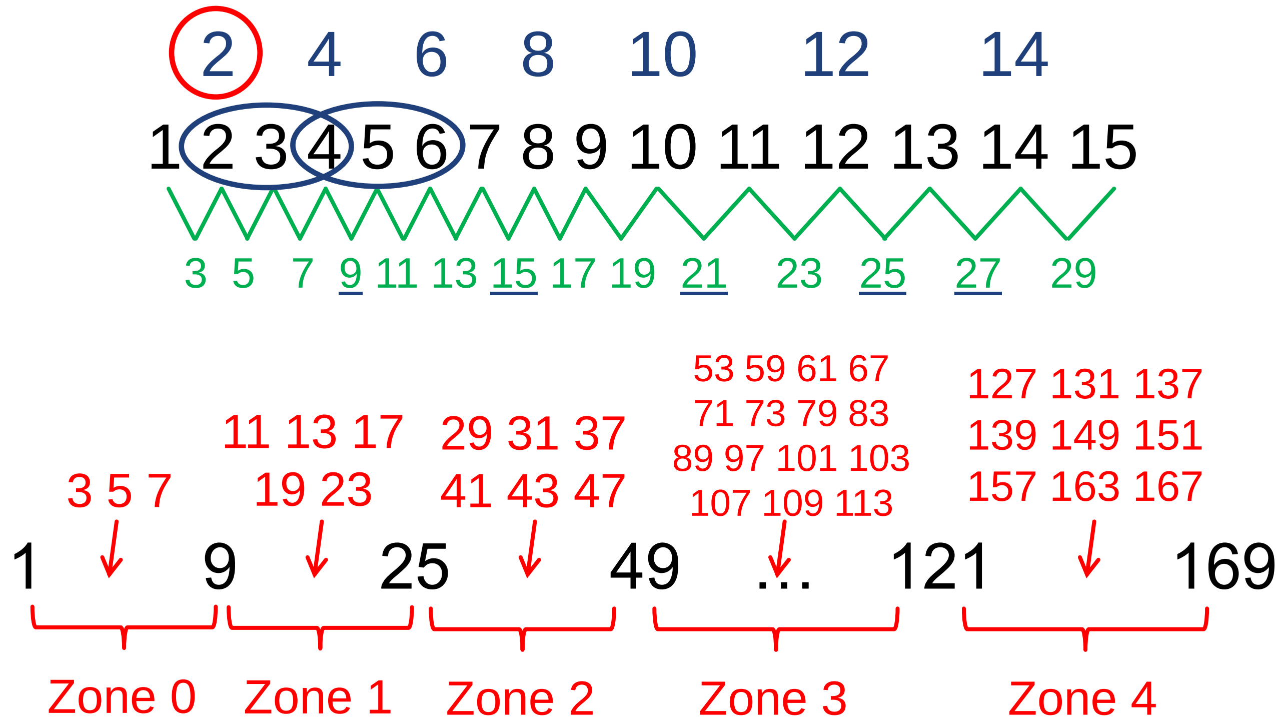
http://www.mscs.dal.ca/~joerg/pic/g-letter.jpg

http://de.wikipedia.org/wiki/Goldbachsche_Vermutung



Quelle: J. Waldvogel

Primes & Composites



1-100:	25 primes	25 composites
1-1000:	167 primes	332 composites

Composite Equation 1

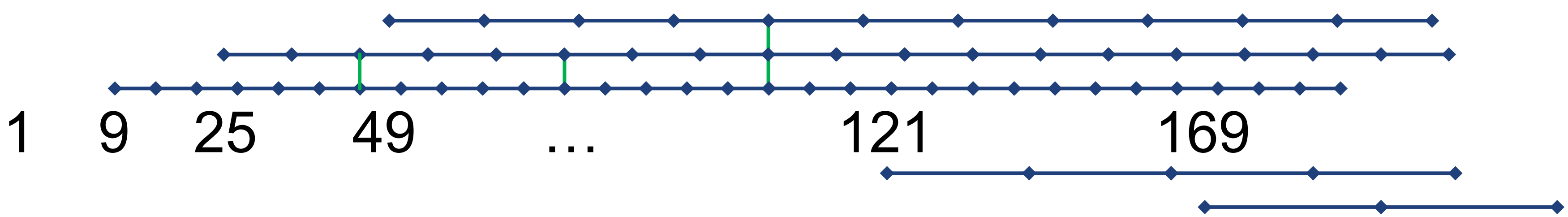
$$x = n^2 + 2nm$$

$n = 3,5,7, \ldots$

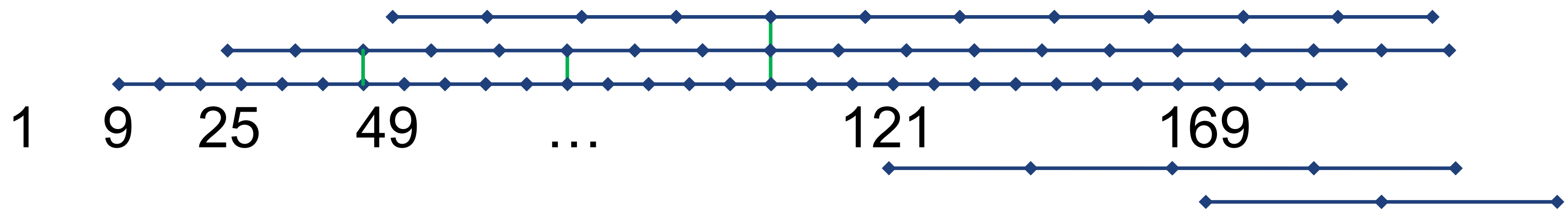
or $n = p$

$m = 0,1,2,3, \ldots$

infinite number of arithmetic series



Composite Equation 1



9 15 21 27 33 39 45 51 57 63 69 75 …

25 35 45 55 65 75 85 95 …

49 63 77 91 105 …

121 143 165 187 209 …

169 …

Composite Equation 2

$$x = n^2 + 2nm$$
$$n = 3,5,7,\ldots$$
$$m = 0,1,2,3,\ldots$$
$$n \to 2n + 3$$
$$n, m = 0,1,2,3,\ldots$$

$$x = \begin{bmatrix} n & m & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 6 \\ 2 & 0 & 3 \\ 6 & 3 & 9 \end{bmatrix} \begin{bmatrix} n \\ m \\ 1 \end{bmatrix}$$

Composite Equation 3

$$x = n^2 + 2nm$$

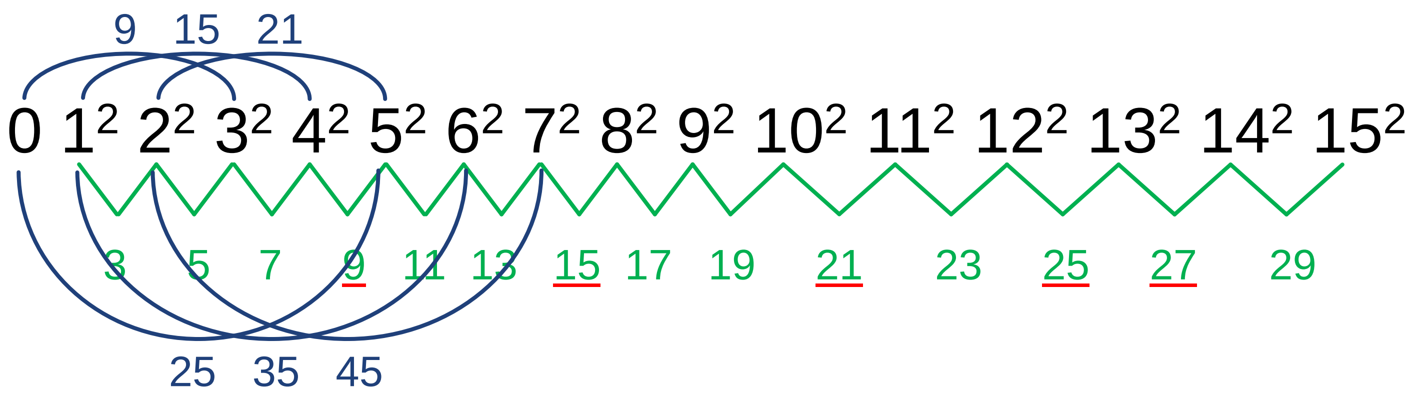
$$\rightarrow n = -m + \sqrt{m^2 + x}$$

- if $m = \frac{x-1}{2}$ then $n=1$
  same for primes & composites

- if $n = 3 \rightarrow m = \frac{x-9}{6}$
  if $n = 5 \rightarrow m = \frac{x-25}{10}$
  if $n = p \rightarrow m = \frac{x-p^2}{2p}$

$$\rightarrow \frac{x}{2p} \text{ for } x \rightarrow \infty$$

Squares



All the composite numbers are differences between squares with distance ≥ 3.

All odd numbers (primes or composites) are differences between neighbouring squares.

Square Equation

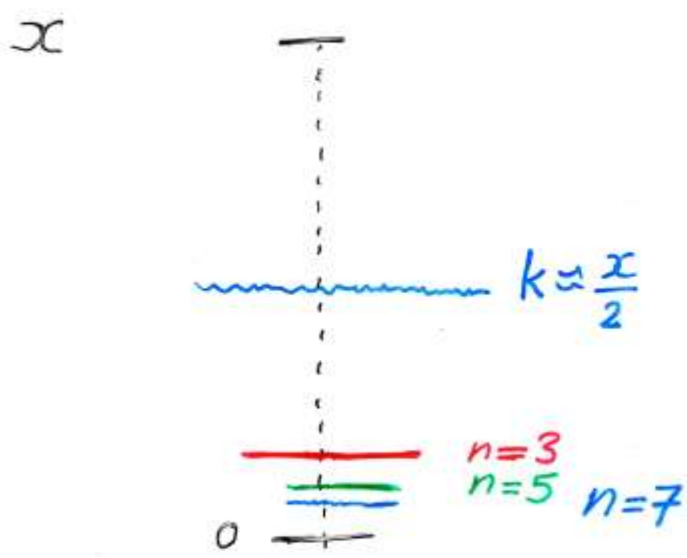
$$x + k^2 = \alpha^2$$

$x = n^2 + 2nk$ $\quad n = 3,5,7,\dots$

$n = 3$ $\quad k \to \frac{x}{6}$

$n = 5$ $\quad k \to \frac{x}{10}$

$n = 7$ $\quad k \to \frac{x}{14}$



Prime Numbers

- Largest: in Jan. 2013 $2^p - 1$
  $2^{57`885`167} - 1$ Mersenne 48
  17’425’170 digits
  using 360’000 CPU at 150 trillion calculations/sec
- Mersenne 47 in 2008
  $2^{43`112`609} - 1$
  12’978’189 digits
- Great Internet Mersenne Prime Search GIMPS 1996
- 1951 79 digits
  Electronic Computer Camridge

Mersenne Prime

$2^n - 1$

n prime

2 3 5 7 13 17 19 31 61 89 107 127 521 607 1279

2203 2281 3217 4253 4423 9689 9941 11213

19937 21701 23209 44497 86243 110503 132049

216091 756839 859433 1257787 1398269

2976221 3021377 6972593 13466917 20996011

24036583 25964951 30402457 | 32582657

37156667 4264380 57885161

Year 2013

Total 48

Fermat Prime

$$2^{2^n} + 1$$

$$F_0 = 3$$

$$F_1 = 5$$

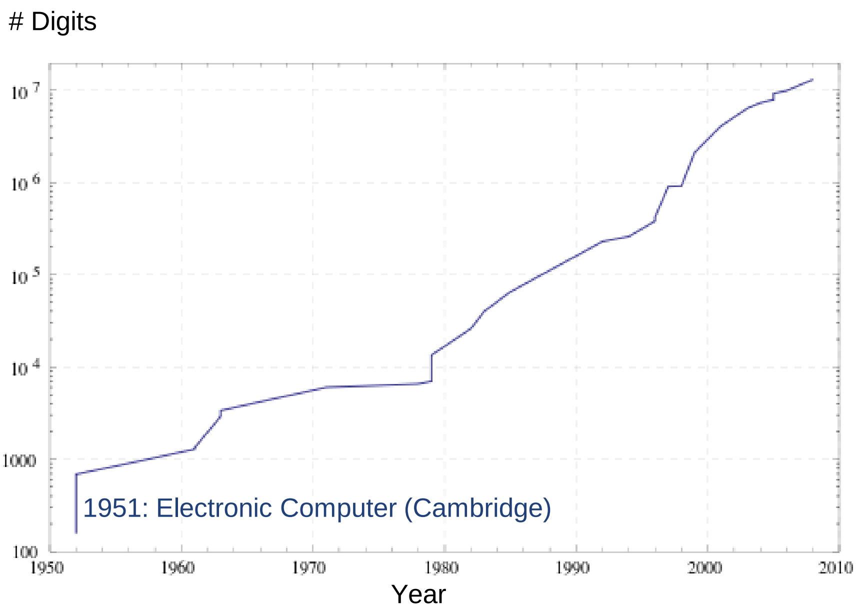
$$F_2 = 17$$

$$F_3 = 257$$

$$F_4 = 65537$$

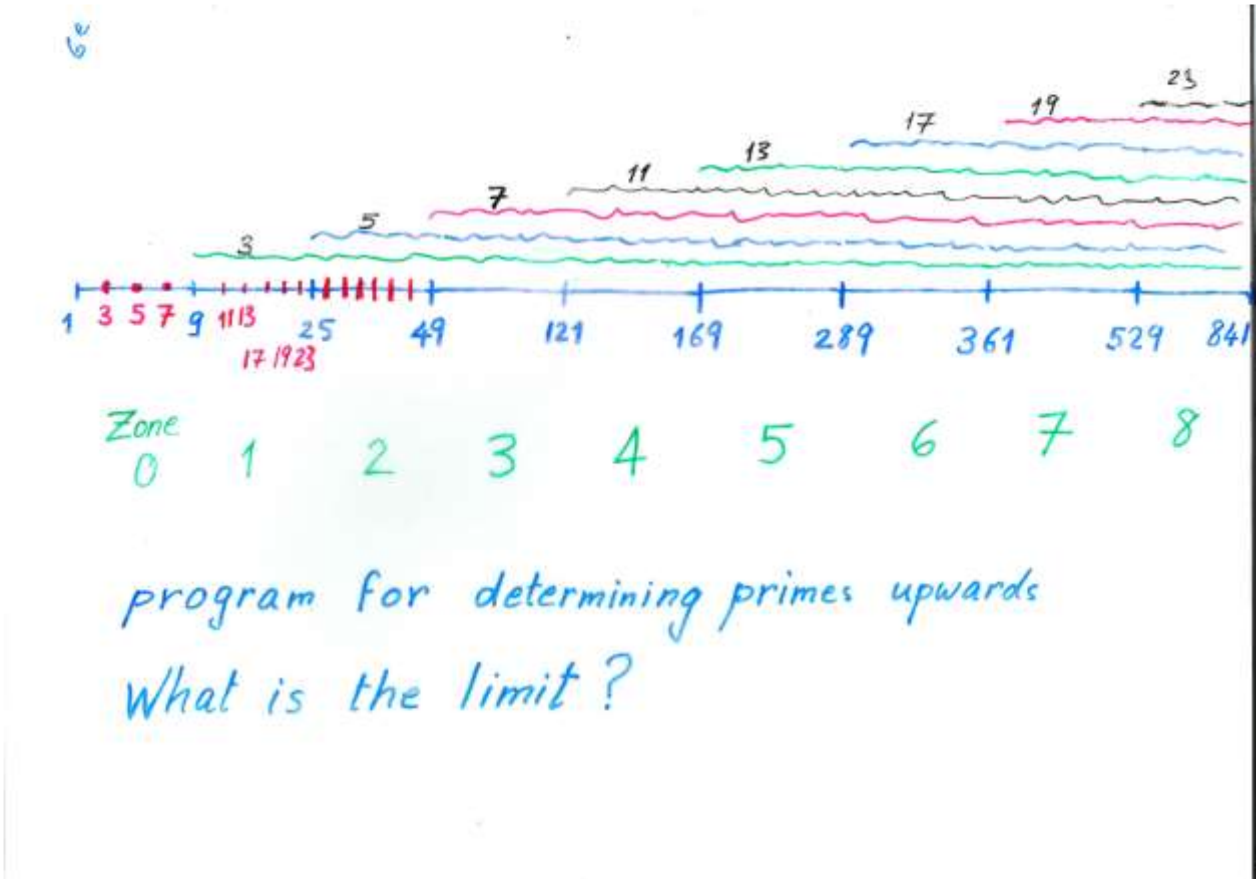
No. Of digits

M_8 Euler 10 digits in 1772



http://en.wikipedia.org/wiki/Mersenne_prime$
By Bender2k14

What is the limit?



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Wilson Theorem & its Extension

- p is prime iff

$$\underline{(p-1)! + 1 = 0\ (mod\ p)}$$

Example p=5 1x2x3x4+1=25

- p is prime iff

$$2,3,4,\ldots,\frac{p-1}{2}$$

are composed of pairs n,m such that

$$nm \pm 1 = 0\ (mod\ p)$$

Example p=19 p=17

2 3 4 5 6 7 8 9 2 3 4 5 6 7 8

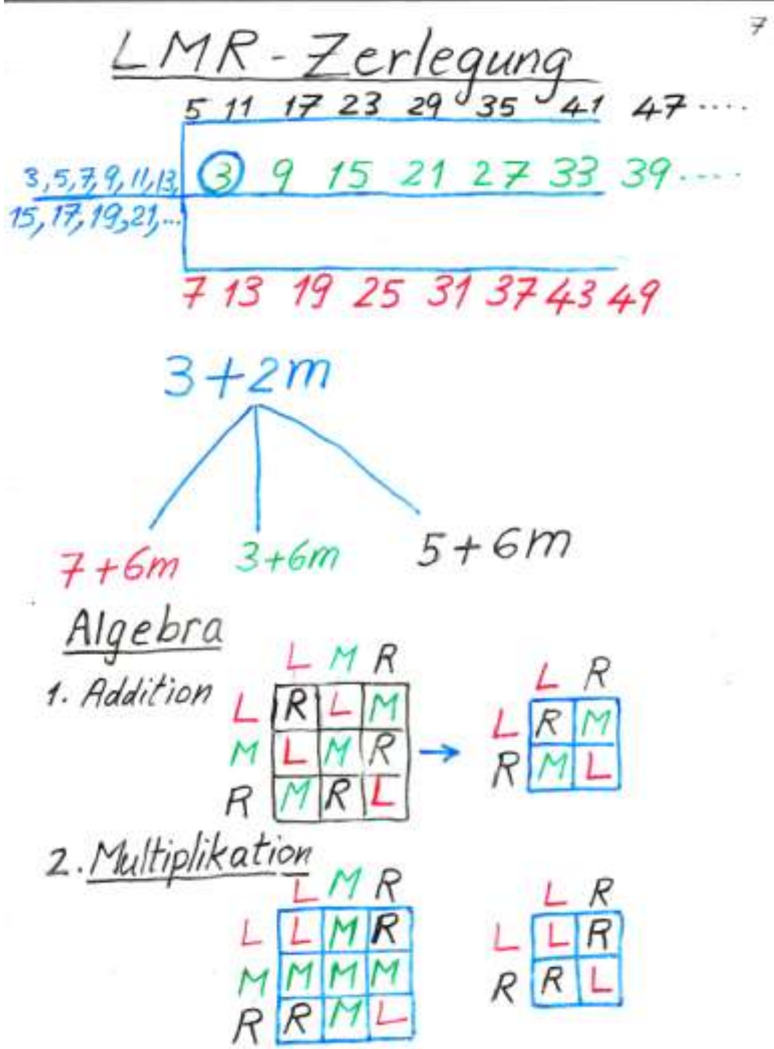
Prime Number Theorem

$$Number\ of\ primes < x$$

$$\frac{x}{\ln x}\ as\ x \to \infty$$

Bessere Approximationen
für x beschränkt

LMR Zerlegung



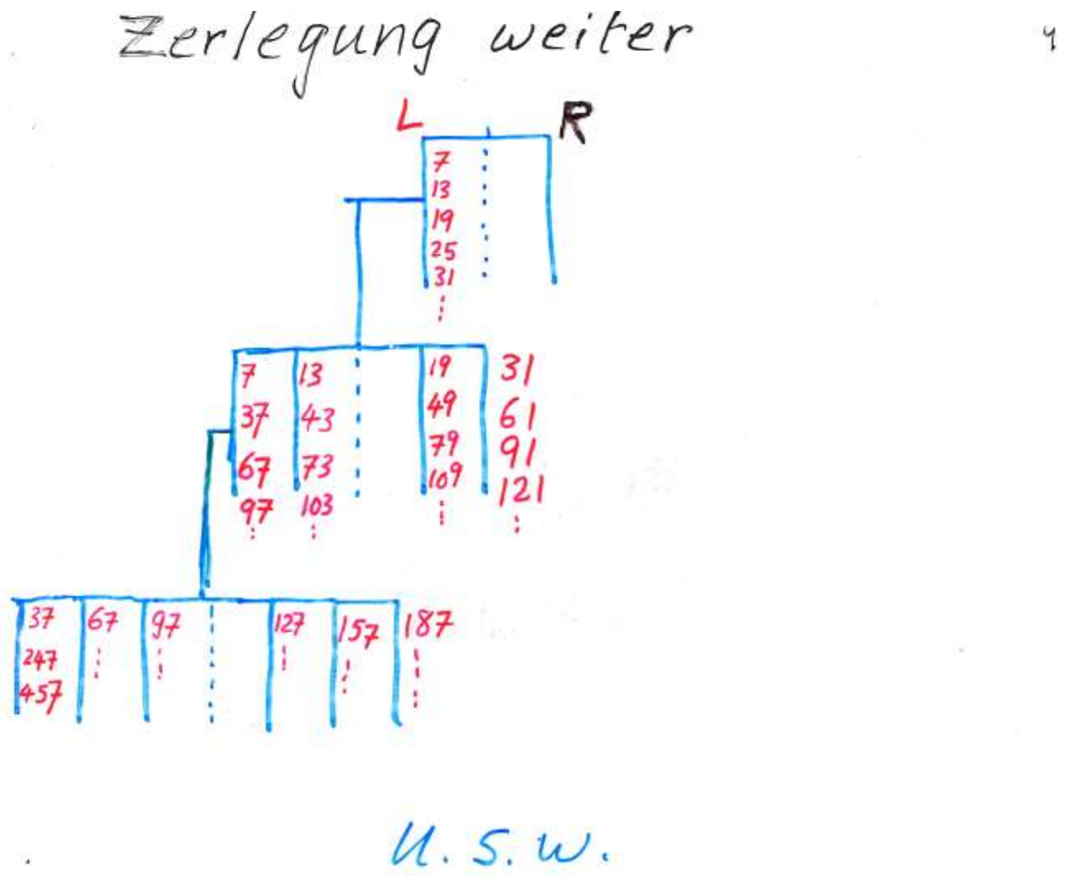
LMR & Goldbach

R even $\rightarrow L + L\ primes$
$or\ 3 + R\ prime$

L even $\rightarrow R + R\ primes$
$or\ 3 + L\ prime$

M even $\rightarrow L + R\ primes$

Zerlegung weiter



Composites in L & R

L

7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97

5 composites & 11 primes

$$x = n^2 + 6nk$$

R

5 11 17 23 29 35 41 47 53 59 65 71 77 83 89 95

4 composites & 12 primes

$$x = n^2 \begin{Bmatrix} +2n \\ +4n \end{Bmatrix} + 6nk$$

2 wenn n R

4 wenn n L

Prime Examination using Quadratic Expression & RL Decomposition

Given x

$$k^2 + x$$

Restrictions

1) Squares can not end with 2, 3, 7, 8
2) For $x\ L \rightarrow k = 0,3,6,9,12,\ldots$
   For $x\ R \rightarrow k = 1,2,4,5,7,8,\ldots$
3) $k \leq \frac{x-49}{14}$

Example x=187 L

3) $\rightarrow \frac{x-49}{14} = 9,\ldots$

2) $\rightarrow k = 0,3,6,9$

1) $\rightarrow k = 3$

$$187 + 9 = 196 = 14^2$$

$$\rightarrow x\ composite$$

SOS & NSOS Decomposition

SOS = sum of squares

odd $x = a^2 + b^2$

a odd & b even

a & b have no common factors

- SOS: 5, 13, 17, 25, 29, 37, 41, 53, 61, 65, 73, 85, 89, 97

  11 primes & 3 composites
- A commutative semi-group closed under multiplication
- The primes are the generating set.
- x is SOS iff 2x is sum of odd squares

SOS & NSOS Decomposition

Another Decomposition

$$3 + 4k$$
$$5 + 4k$$

Used in getting some information on factorization

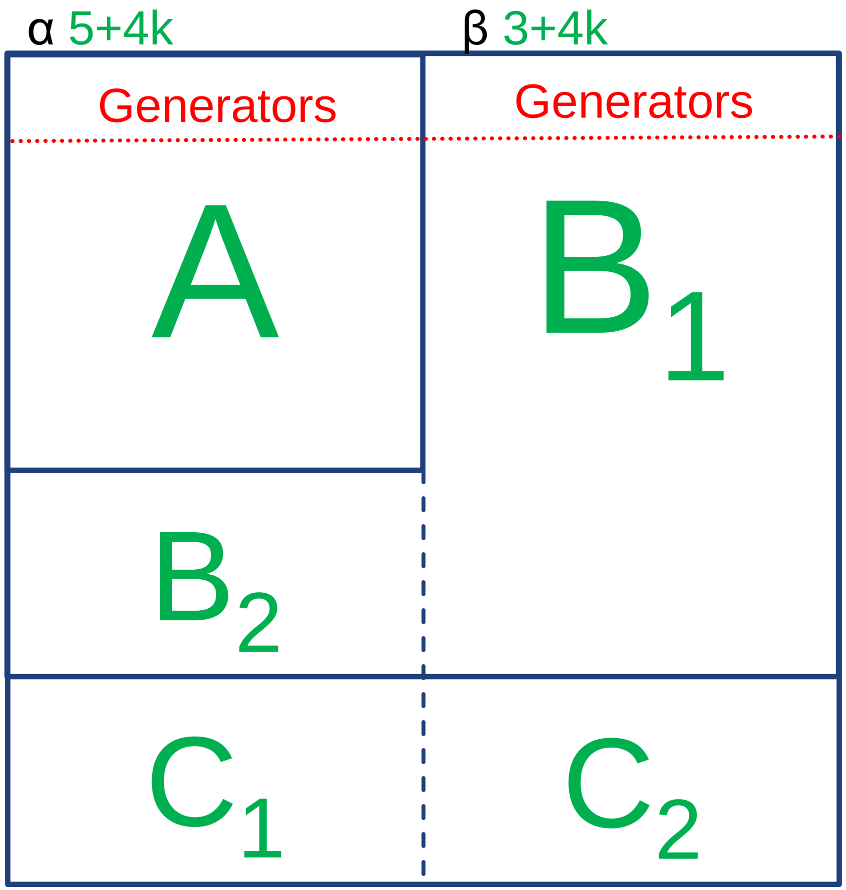
Characteristics of SOS & NSOS

All the SOS numbers are of the form $5 + 4k$ (A)

The NSOS numbers are either of the form $3 + 4k$ (B_1)
or of the form $5 + 4k$ (B_2)

All the cross multiplication numbers are either $3 + 4k$ (C_1)
or $5 + 4k$ (C_2)

All the NSOS primes are of the form $3 + 4k$ (in B_1)


α 5+4k
β 3+4k
Generators
Generators
A
B1
B2
C1
C2

Remark 1

- Sum of odd squares is always the Set A multiplied by 2 (a,b are either coprime or have common factors which are SOS)
- Sum of even squares under the same condition is 2i times a SOS number i>1

$$1^2 + 3^2 = 2 \cdot 5$$
$$10^2 + 20^2 = 2 \cdot 10 \cdot 25$$

Remark 2

- x is in A if
  i. x is expressed as $5 + 4k$
  ii. x should not have NSOS prime factors $< \sqrt{x}$
- Effort is approximately half the effort for proving primeness

Goldbach & the Gaps

3 5 7 · 11 13 · 17 19 · 23 · · 29 31 · · 37 · 41 43 · 47 · · 53 ·
· 59 61 · · 67 · 71 73 · · 79 · 83 · · 89 · · · 97 · 101 103 ·
107 109 · 113 · · · · · · 127 · 131

Add 3 & use the other primes in the gaps

Problem numbers inside the gap are given by

$$x = n^2 + 2nk - 3$$

95	119	125
↓	↓	↓
98	122	128

This can be proved using L

Using L

7 13 19 · 31 37 43 · · 61 67 73 79 · · 97 103 109 · · 127 · 139 · 151 157 163 · · 181 · 193 199 · 211 · 223 229 · 241 · · · · 271 277 283 · · · 307 313 · · 331 337 · 349 · · 367 373 379 · · 397 · 409 · 421 · 433 439 · · 457 463 · · · 487 · 499 · · · 523 · · 541 547 · · · 571 577 · · · 601 607 613 649 · 631 · 643 · · 661 · 673 · · 691 · · 709 · · 727 733 739 · 751 757 · 769 · · 787 · · · 811 · 823 829 · · · 853 859 · · 877 883 · · · 907 · 919 · · 937 · · · · 967 · · · 991 997

L Extension 1

7	37	67	97	127	157	187	…
13	43	73	103	133	…		
19	49	79	109	139	…		
31	61	91	121	151	…		

524	824			
266				
308	488	908		
572	602	812	962	992

The Reflected Function
Goldbach Equivalent 1

0 X even

$\overrightarrow{x \equiv} n^2 + 2nk$

$y = X - \overleftarrow{n^2 - 2nk}$

If y does not go through all the primes then Goldbach is valid

First order example

$\overrightarrow{x \equiv} 3 + 4k$

$y = 100 - \overleftarrow{3 - 4k}$

goes through all the values which x does not go through

The Quadratic Expression Goldbach Equivalent 2

$k^2 + x$ $\mathrm{X} = 2x$

x either composite or even

Does ε exist s.t.

$x + \varepsilon$ & $x - \varepsilon$ are primes

(ε is even in case x odd, ε is odd in case x even)

- Prime is controlled by the above expression
- x odd is L or R $\varepsilon = 6\varepsilon_1$
- x even $\varepsilon = 3\varepsilon_1$ & ε_1 odd

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Vielen Dank für Ihre Aufmerksamkeit.